8.1 Exercise Set

1. Fill in the blanks.

   a) The domain of a sequence is the set of consecutive natural numbers.

   b) A sequence with a last term is a(n) finite sequence.

   c) A sequence with no last term is a(n) infinite sequence.

   d) The sequence $a_1 = 2,\ a_n = 2a_{n-1}$ is a recursive sequence.

   e) The formula for the nth term of an arithmetic sequence is $t_n =$ $a+(n-1)d$

2. Write the first four terms of each sequence.

   a) $\{n^2 - 2\}$
   $-1, 2, 7, 12$

   b) $\left\{\frac{n+2}{n+1}\right\}$

   c) $\{(-1)^{n+1}n^2\}$
   $1, -4, 9, -16$

   d) $\left\{\frac{3^n}{2^n+1}\right\}$

   e) $\left\{\frac{2^n}{n^2}\right\}$
   $2, 1, 8/9, 1$

   f) $\left\{\left(\frac{2}{3}\right)^n\right\}$

3. Write the nth term of the suggested pattern.

   a) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ $\frac{1}{n}$

   b) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

   c) $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots$ $\frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n$

   d) $2, -4, 6, -8, \ldots$

4. Write the first four terms of the recursive sequence.

   a) $a = 4,\ t_n = 2 + t_{n-1}$
   $4, 6, 8, 10$

   b) $a = 3,\ t_n = n - t_{n-1}$

   c) $a = 2,\ a_2 = 3,\ a_n = a_{n-1} + a_{n-2}$
   $2, 3, 5, 8$

   d) $a_1 = -1,\ a_2 = 1,\ a_n = na_{n-1} + a_{n-2}$

5. Find the sum of each sequence.

a) $\sum_{k=1}^{5} 4 = 4$

b) $\sum_{k=1}^{4}(k^2-2)$

c) $\sum_{k=2}^{5}(k^2-1) = 50$

$2^2-1+3^2-1+4^2-1+5^2-1$

d) $\sum_{k=0}^{3}(k^3-1)$

e) $\sum_{k=1}^{4}\frac{k^2}{2} = 15$

$\frac{1}{2}+\frac{4}{2}+\frac{9}{2}+\frac{16}{2}$

f) $\sum_{k=6}^{8}(k+1)^2$

6. Express each sum using summation notation with index $k = 1$.

a) $1+3+5+7$ $\sum_{k=1}^{4}(2k-1)$

b) $1^2+2^2+3^2+4^2+5^2$

c) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots+\frac{n}{n+1}$ $\sum_{k=1}^{n}\frac{k}{k+1}$

d) $5+\frac{5^2}{2}+\frac{5^3}{3}+\cdots+\frac{5^n}{n}$

7. Write the first five terms of each arithmetic sequence. common d.

a) $7, 11, 15, \underline{19}, \underline{23}$ (+4)

b) $15, 12, 9, \_\_\_\_, \_\_\_\_$

c) $a = 4,\ d = 2$

$4, 6, 8, 10, 12$

d) $a = -1,\ d = -3$

e) $a = -5,\ d = -\frac{3}{4}$

$-5, \frac{-23}{4}, \frac{-13}{2}, \frac{-29}{4}, -8.$

f) $a = -\frac{2}{3},\ d = \frac{1}{5}$

8. Find the indicated arithmetic term.

a) $a = 5,\ d = 3$; find t_{12}

$t_{12} = 5+(11)(3)$

$= 38$

b) $a = \frac{2}{3},\ d = -\frac{1}{4}$; find t_9

c) $a = -\frac{3}{4},\ d = \frac{1}{2}$; find t_{10}

$t_{10} = -\frac{3}{4}+(9)(1/2) = 15/4$

d) $a = 2.5,\ d = -1.25$; find t_{20}

e) $a = -0.75,\ d = 0.05$; find t_{40}

$t_{40} = -0.75+(39)(0.05)$

$= 1.2$

f) $a = -1\frac{3}{4},\ d = -\frac{2}{3}$; find t_{37}

9. Find the number of terms in each arithmetic sequence.

a) $a = 6,\ t_n = -30,\ d = -3$

$-30 = 6 + (n-1)(-3)$

$-36 = (n-1)(-3)$

$12 = n-1 \qquad n = 13$

b) $a = -3,\ t_n = 82,\ d = 5$

c) $a = 0.6,\ t_n = 9.2,\ d = 0.2$

$9.2 = 0.6 + (n-1)(0.2)$

$8.6 = (n-1)(0.2) \qquad n = 44.$

$43 = n-1$

d) $a = -0.3,\ t_n = -39.4,\ d = -2.3$

e) $-1, 4, 9, \ldots, 159$

$159 = -1 + (n-1)(5)$

$160 = (n-1)5.$

$32 = n-1 \qquad n = 33$

f) $23, 20, 17, \ldots, -100$

10. Find the first term in the arithmetic sequence.

$t_6 = 10 = a + 5(3) \qquad a = -5$

a) 6th term is 10; 18th term is 46

$10 + 12d = 46$

$12d = 36$

$d = 3.$

b) 4th term is 2; 18th term is 30

c) 9th term is 23; 17th term is − 1

$23 + 8d = -1 \qquad 23 = a + 8(-3)$

$8d = -24 \qquad 47 = a$

$d = -3$

d) 5th term is 3; 25th term is − 57

e) 13th term is − 3; 20th term is − 17

$-3 + 7d = -17 \qquad -3 = a + (12)(-2)$

$7d = -14 \qquad a = 21$

$d = -2$

f) 11th term is 37; 26th term is 32

11. Find x so that the values given are consecutive terms of an arithmetic sequence.

a) $x + 3,\ 2x + 1$, and $5x + 2$

$\frac{x+3+5x+2}{2} = 2x+1$

$6x + 5 = 4x + 2$

b) $2x,\ 3x + 2$, and $5x + 3$

c) $x - 1,\ \frac{1}{2}x + 4$, and $1 - 2x \qquad 2x = -3 \qquad x = -\frac{3}{2}$

$\frac{x-1+1-2x}{2} = \frac{1}{2}x + 4 \qquad -x = x + 8$

$-8 = 2x$

$x = -4$

d) $2x - 1,\ x + 1$, and $3x + 9$

e) $x + 4,\ x^2 + 5$, and $x + 30$

$\frac{x+4+x+30}{2} = x^2 + 5$

$2x + 34 = 2x^2 + 10$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4, -3$

f) $8x + 7,\ 2x + 5$, and $2x^2 + x$

12. If t_n is a term of an arithmetic sequence, what is $t_n - t_{n-1}$ equal to? the difference

13. List the first seven numbers of the Fibonacci sequence $a_1 = 1,\ a_2 = 1,\ a_n = a_{n-1} + a_{n-2},\ n > 2$.

14. The starting salary of an employee is \$23 750. If each year a \$1250 raise is given, in how many years will the employee's salary be \$50 000?

$50\,000 = 23750 + (n-1)(1250)$

$n = 22$ yrs.

15. An auditorium has 8 seats in the first row. Each subsequent row has 4 more seats than the previous row. What row has 140 seats?

16. A well drilling company charges \$8.00 for the first meter, then \$8.75 for the second meter, and so on in an arithmetic sequence. At this rate, what would be the cost to drill the last meter of a well 120 meters deep?

$t_{120} = 8 + (119)(0.75)$

$= \$97.25$

17. It is said that during the last weeks of his life Abraham deMoivre needed 15 minutes more sleep each night, and when he needed 24 hours sleep he would die. If he needed 8 hours sleep on September 1, what day did he die?

18. The first three terms of an arithmetic sequence are $x - 3$, $\frac{x^2}{25} + 9$, and $3x - 11$. Determine the fourth term.

$\frac{x-3+3x-11}{2} = \frac{x^2}{25} + 9$

$4x - 14 = \frac{2x^2}{25} + 18$

$\left[0 = \frac{2x^2}{25} - 4x + 32\right]25$

19. The first, third, and fifth terms of an arithmetic sequence are $2x - 1$, $x^2 - 3$, and $11 - x^2$ respectively. Determine the second term.

$0 = 2x^2 - 100x + 800$

$0 = x^2 - 50x + 400$

$0 = (x-40)(x-10)$

$x = 40,\ 10.$

1) 37, 73, 109, 145

2) 7, 13, 19, $\frac{145}{25}$

$S_n = \frac{n}{2}(a+l) = \frac{n}{2}(2a+(n-1)d)$

8.2 Exercise Set

1. Find the sum of the arithmetic series.

a) $3 + 5 + 7 + \cdots + (2n + 1)$

$S_n = \frac{n}{2}(3+2n+1) = \frac{n}{2}(4+2n)$

$= \frac{4n+2n^2}{2} = 2n+n^2$

b) $-1 + 2 + 5 + \cdots + (3n - 4)$

$S_n = \frac{n}{2}(-1+3n-4) = \frac{n}{2}(3n-5)$

$= \frac{3n^2-5n}{2}$

c) $2 + 5 + 8 + \cdots + 77$

$77 = 2+(n-1)(3) \quad n=26$

$S_{26} = \frac{26}{2}(2+77) = 1027$

d) $5 + 9 + 13 + \cdots + 97$

e) $(-41) + (-35) + (-29) + \cdots + 541$

$541 = -41+(n-1)(6) \quad n=98$

$S_{98} = \frac{98}{2}(-41+541) = 24\,500$

f) $2\sqrt{5} + 6\sqrt{5} + 10\sqrt{5} + \cdots + 50\sqrt{5}$

g) $39 + 33 + 27 + \cdots + (-15)$

$-15 = 39+(n-1)(-6) \quad n=10$

$S_{10} = \frac{10}{2}(39-15) = 120$

h) $23 + 19 + 15 + \cdots + (-305)$

i) $\frac{1}{2} + \frac{7}{8} + \frac{5}{4} + \cdots + \frac{55}{8}$

$\frac{55}{8} = \frac{1}{2}+(n-1)\left(\frac{3}{8}\right) \quad n=18$

$S_{18} = \frac{18}{2}\left(\frac{1}{2}+\frac{55}{8}\right) = \frac{531}{8}$

j) $\frac{16}{3} + \frac{13}{3} + \frac{10}{3} + \cdots + \left(-\frac{65}{3}\right)$

k) $3.7 + 9 + 14.3 + \cdots + 30.2$

$30.2 = 3.7+(n-1)(5.3) \quad n=6$

$S_6 = \frac{6}{2}(3.7+30.2) = 101.7$

l) $2.84 + 5.3 + 7.76 + \cdots + 79.1$

2. Find the indicated value using the information given.

a) S_{20}, if $a_1 = 8,\ a_{20} = 65$

$65 = 8 + (20-1)d,\ d = 3$

$S_{20} = \frac{20}{2}(2(8) + (20-1)(3))$

$S_{20} = 730$

b) S_{21}, if $a_1 = 8,\ a_{20} = 65$

c) S_{56}, if $a_{56} = 13,\ d = -9$

$13 = a + (56-1)(-9)$

$a = 508$

$S_{56} = \frac{56}{2}(2(508) + (56-1)(-9))$

$S_{56} = 14\,588$

d) n, if $S_n = 180,\ a_1 = 4,\ a_n = 16$

e) d, if $S_{40} = 680,\ a_1 = 11$

$S_{40} = \frac{40}{2}(2(11) + (40-1)d) = 680$

$S_{40} = 20(22 + 39d) = 680$

$d = 4/13$

f) S_{62}, if $a_1 = 10,\ d = 3$

g) S_{19}, if $d = 4,\ a_{19} = 17$

$17 = a + (19-1)(4)\quad a = -55$

$S_{19} = \frac{19}{2}(2(-55) + (19-1)(4))$

$S_{19} = -361$

h) S_{40}, if $d = -3,\ a_{40} = 65$

i) S_{40}, if $a_5 = 42,\ a_{15} = -18$

$42 + 10d = -18$

$10d = -60$

$d = -6$

j) S_{20}, if $a_8 = 17,\ a_{15} = 38$

$42 = a + (5-1)(-6)$

$a = 66$

$S_{40} = \frac{40}{2}(2(66) + (40-1)(-6))$

$S_{40} = -2040$

3. Find the indicated sum.

a) $\sum_{n=1}^{100} n$ $a=1$, $t_2=2$ $\Rightarrow d=1$, $n=100$

b) $\sum_{k=100}^{200} k$

$S_{100}=\frac{100}{2}(2(1)+(100-1)(1))$

$S_{100}=5050$

c) $\sum_{j=0}^{72}(3j-4)$ $a=-4$, $t_2=-1$ $\Rightarrow d=3$, $n=73$

d) $\sum_{x=7}^{24}(2x+5)$

$S_{72}=\frac{73}{2}(2(-4)+(73-1)(3))$

$S_{72}=7592$

e) $\sum_{y=11}^{48}\left(\frac{y+4}{2}\right)$ $a=7.5$, $t_2=8$ $\Rightarrow d=0.5$

$n=48-11+1=38$

$S_{38}=\frac{38}{2}(7.5(2)+(38-1)(0.5))$

$S_{38}=636.5$

f) $\sum_{z=51}^{100}(200-z)-\sum_{z=1}^{50}(200-z)$

4. Insert k arithmetic means between the given pair of numbers.

a) 5, 10, $k=2$

$5+3d=10$

$3d=5$

$d=5/3$

$5\ \frac{20}{3}\ \frac{25}{3}\ 10$

b) 3, 6, $k=3$

c) a, b, $k=2$

$a+3d=2$

$3d=2-a$

$d=\frac{2-a}{3}$

$a\ \frac{2a+2}{3}\ \frac{a+4}{3}\ 2$

$\frac{3a}{3}+\frac{2-a}{3}=\frac{2a+2}{3}+\frac{2-a}{3}$

d) a, b, $k=3$

5. Solve for b: $\sum_{x=2}^{b}(23-2x)=91$

6. Find the sum: $\sum_{x=a}^{b}5$

$91=\frac{b-1}{2}(23-2(2)+23-2b)$

$91=\frac{b-1}{2}(42-2b)$

$182=(b-1)(42-2b)$

$182=42b-42-2b^2+2b$

$2b^2-44b+224=0$

$b^2-22b+112=0$

$(b-14)(b-8)=0$

$b=14, 8$

7. What is the last element in the 20th row?

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          1
        2   3
      4   5   6
    7   8   9   10
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$1, 1+2, 1+2+3, 1+2+3+4$

$20^{th}: 1+2+\cdots+20$

$S_{20} = \frac{20}{2}(1+20) = 210$

8. How many terms of the arithmetic series $1491 + 1484 + 1477 + \cdots$ are needed to give a sum of zero?

9. An auditorium has eight seats in the first row. Each subsequent row has four more seats than the previous row. How many seats are there in the 50th row of the auditorium?

$t_{50} = 8 + 49(4) = 204$.

10. If \$1000 is deposited into the bank the day a child is born, and \$100 more than the previous deposit is made each year until the child's 18th birthday, how much will be in the account, excluding interest?

11. Find the sum of all multiples of 6 between 50 and 500.

$6 \times 9 = 54$

$6 \times 83 = 498$.

$498 = 54 + (n-1)6 \quad n = 75$

$S_{75} = \frac{75}{2}(54 + 498) = 20700$

12. The sum of three consecutive terms of an arithmetic sequence is 3. The sum of their squares is 75. Find the three numbers.

13. If 20 people in a class shake hands with each other exactly once, how many handshakes will take place?

$19 + 18 + 17 \cdots 1$

$S_{19} = \frac{19}{2}(19 + 1) = 190$.

14. If the sum of the terms of an arithmetic series is 234, and the middle term is 26, find the number of terms in the series.

8.3 Exercise Set

1. Determine if the sequence is geometric. If it is, find the common ratio.

a) 4, 12, 36, 72,... no.

b) 3, 12, 48, 142,... no.

c) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$ yes $r=-1/2$

d) $1, -1, 1, -1, \ldots$ yes $r=-1$

e) $3, -6, -12, 24, \ldots$ no.

f) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ no

g) $\frac{1}{4}, \frac{1}{6}, \frac{1}{9}, \frac{2}{27}, \ldots$ yes $r=2/3$

h) $\frac{2}{5}, -\frac{2}{3}, \frac{10}{9}, -\frac{50}{27}, \ldots$ yes $r=-5/3$

i) $3x^2, 12x^4y^3, 48x^6y^6, \ldots$ yes $r=4x^2y^3$

j) $\sqrt{2}, \sqrt{6}, 3\sqrt{2}, 3\sqrt{6}, \ldots$ yes $r=\sqrt{3}$

2. Write the first five terms of the geometric sequence.

a) 1, 4, 16, 64, 256

b) 1, ____, 4, ____, ____

c) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

d) 4, ____, ____, − 13.5, ____

e) 162, 54, 18, 6, 2

f) 1, ____, 3, ____, ____

g) $3, 3^{2x+1}, 3^{4x+1}, 3^{6x+1}, 3^{8x+1}$

h) 1, ____, x^4, ____, ____

i) $5, 5^{2x-1}, 5^{4x-3}, 5^{6x-5}, 5^{8x-7}$

j) $1, -\frac{x}{3}$, ____, ____, ____

3. Find all possible values of r for a geometric sequence with the two given terms.

a) $a_5 = 5, a_7 = 25$

b) $a_2 = 4, a_6 = \frac{1}{4}$

c) $a_4 = 2\sqrt{2}, a_7 = 8$

d) $a_3 = 1, a_6 = \sqrt{2}$

4. Find the indicated value using the information given.

a) a_{11}, if $a_1 = \frac{1}{128}$, $r = 2$

$t_{11} = \frac{1}{128}(2)^{10}$

$t_{11} = \boxed{8}$

b) a_9, if $a_1 = 3$, $a_2 = \sqrt{3}$

c) a_{42}, if $a_{40} = 9$, $a_{41} = 36$

$t_{41} = 36 = ar^{40}$

$t_{40} = 9 = ar^{39}$

$4 = r$

$t_{42} = 36 \times 4 = \boxed{144}$

d) a_9, if $a_4 = 5$, $a_6 = 20$

e) n, if $a_1 = 729$, $a_2 = 243$, $l = \frac{1}{9}$

$r = \frac{243}{729} = \frac{1}{3}$

$t_n = 729(1/3)^{n-1} = \frac{1}{9}$

$(\frac{1}{3})^{n-1} = \frac{1}{6561}$

$n - 1 = 8$

$n = \boxed{9}$

f) n, if $a_1 = 2048$, $a_2 = 1024$, $l = 1$

g) a_1, if $a_5 = 27$, $r = 3$

$t_5 = 27 = a(3)^4$

$\boxed{\frac{1}{3}} = \frac{27}{81} = a$

h) a_1, if $a_7 = 128$, $r = 4$

i) r, if $a_{10} = 25$, $a_{12} = 225$

$t_{12} = 225 = ar^{11}$

$t_{10} = 25 = ar^9$

$9 = r^2$ $r = \boxed{\pm 3}$

j) r, if $a_{25} = 12$, $a_{31} = 96$

k) a_8, if $a_n = 3a_{n-1}$, $a_1 = \frac{1}{27}$

$a_2 = 3(\frac{1}{27}) = \frac{1}{9}$

$\frac{a_2}{a_1} = r = 3$

$a_8 = \frac{1}{27}(3)^7 = \boxed{81}$

l) a_6, if $a_n = 0.1a_{n-1}$, $a_1 = 1000$

5. Insert two geometric means between a and b.

$a \_\_ \_\_ b.$

$a \quad ar \quad ar^2 \quad ar^3.$

$b = ar^3. \qquad r = \sqrt[3]{\frac{b}{a}}$

$\frac{b}{a} = \frac{ar^3}{a}$

6. Given the geometric sequence $a, \frac{a}{b}, \frac{a}{b_2}, \ldots$ determine an expression for $t_n - t_{n-1}, \; n > 2$.

7. Find x so that $x - 1, \; x$, and $x + 2$ are consecutive terms of a geometric sequence.

$\frac{x}{x-1} = \frac{x+2}{x} \; (= r)$

$x^2 = x^2 + x - 2$

$0 = x - 2$

$x = 2$

8. Find the common ratio r for the geometric sequence $x - 2, \; 5 - x, \; 5x - 7, \ldots$

9. What number must be added to $-2, \; 4, \; 19$ so that the resulting numbers are three terms of a geometric sequence?

$-2 + x, \; 4 + x, \; 19 + x.$

$\frac{4+x}{-2+x} = \frac{19+x}{4+x}$

$16 + 8x + x^2 = -38 + 17x + x^2$

$54 = 9x$

$x = 6$

10. If the first two terms of a geometric sequence are $\sqrt{2}$, and $\sqrt[3]{2}$, what is the fourth term?

11. If the product of the first three terms of a geometric is -8, and the sum is $\frac{14}{3}$, what is the common ratio of the sequence?

$a(ar)(ar^2) = -8$

$\sqrt[3]{a^3r^3} = \sqrt[3]{-8}$

$ar = -2$

12. In the sequence $3, \; x, \; y, \; 25$, the first three terms form an arithmetic sequence, and the last three terms form a geometric sequence. Find x and y.